

A Full Rate, Full Diversity Four Antenna Quasi-Orthogonal Space-Time Block Code

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Abstract

We present a complex, full rate space-time block code for four transmit antennas. Using carefully tailored constellation phase rotations, we show this code achieves full diversity for specialized PSK-based constellations. The optimal receiver for the new code decouples the symbol detection problem into pairs of symbols, thus greatly reducing complexity.

1 Introduction

The Alamouti code [1], remarkable for having an elegant and simple linear receiver, became a paradigm in space-time block coding (STBC). Alamouti's idea for 2 transmit antennas was generalized by orthogonal designs [2,3], which have full diversity and simple linear maximum-likelihood (ML) detectors that decouple the transmitted symbols. Unfortunately, the Hurwitz-Radon theorem showed that square, complex, linear processing orthogonal designs cannot achieve full diversity and full rate simultaneously, except in the two transmit antenna case [2,4]. However, recent work

in [5–7] realizes the possibility of designing orthogonal codes over *subsets* of the real or complex plane for more than two transmit antennas.

Prior work on space-time code design includes the ABBA code [8] and similar 4 transmit antenna codes in [9–11] which have full rate, but are quasi-orthogonal and offer a diversity order of only 2. In [12], Papadias’ code [10] is improved using constellation rotations. The result is a quasi-orthogonal full rate and full diversity code for 4 antennas. The STTD-OTD code [13] provides some diversity gain by grouping symbols into Alamouti blocks and transforming them using a Walsh-Hadamard matrix. For the 4 transmit antenna case, this orthogonal code has full rate and diversity order 2. Another code in the literature utilizes space-time diversity with unitary constellation rotating precoders [14–16]. These codes are distinct for achieving full rate and full diversity, but they are not orthogonal and do not afford linear complexity optimal decoding.

In this paper, we propose a new quasi-orthogonal space-time block code for 4 transmit antennas. Using a constellation rotation technique similar to that in [12], we start with a different base code and show that the new code can achieve full rate and full diversity. Performance of the new code with QPSK symbols matches that of [12] with comparable decoding complexity, though the codes are fundamentally different. We conjecture that this performance may represent the limit of what can be achieved for this class of quasi-orthogonal codes.

2 Channel Model

Assume L symbols are sent in each code block of T time intervals. We model a space-time block coded system with N_t transmit and N_r receive antennas as

$$\mathbf{R} = \mathbf{S} \cdot \mathbf{H} + \mathbf{N}, \tag{1}$$

where \mathbf{R} is a $T \times N_r$ matrix representing the received symbols, \mathbf{S} is a $T \times N_t$ matrix representing the transmitted data, \mathbf{H} is an $N_t \times N_r$ matrix representing quasi-static flat fading, and \mathbf{N} is a $T \times N_r$ matrix representing additive white Gaussian noise (AWGN).

A given column of matrix \mathbf{S} represents the complex baseband symbols sent by a specific transmit antenna, while a given row represents the information sent in the

corresponding time interval. The average energy transmitted in each space-time code block satisfies $E[\text{trace}(\mathbf{S}\mathbf{S}^H)] = TN_tE$, where E is the average complex baseband symbol energy and H denotes conjugate transpose.

3 The New Code

Our goal is to design a square, full rate (i.e. $R = \frac{L}{T} = 1$), full diversity 4 transmit antenna space-time block code with M -ary constellations. We start with the orthogonal STTD-OTD code defined in [13] as

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_1 & s_2 & -s_3 & -s_4 \\ -s_2^* & s_1^* & s_4^* & -s_3^* \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{A} & -\mathbf{B} \end{bmatrix}, \quad (2)$$

where \mathbf{A} and \mathbf{B} are Alamouti blocks. This code has diversity order 2 since each symbol passes through only 2 of the 4 transmit antennas. All symbols must be transmitted over every antenna to achieve full diversity, so it is clear a modification of the code matrix is required. We encode the transmitted symbols as follows:

$$s_1 = \frac{d_1+d_2}{\sqrt{2}}, \quad s_2 = \frac{d_3+d_4}{\sqrt{2}}, \quad s_3 = \frac{d_1-d_2}{\sqrt{2}}, \quad s_4 = \frac{d_3-d_4}{\sqrt{2}}, \quad (3)$$

where $d_1, d_2, d_3,$ and d_4 are complex “base” symbols representing the data and $s_1, s_2, s_3,$ and s_4 are complex transmitted symbols. The $\sqrt{2}$ is necessary to normalize energy for symbol constellations centered about the origin of the complex plane. Notice that symbols s_1 and s_3 (similarly s_2 and s_4) are dependent, but have an expanded constellation size that maintains the same rate. The rate of this code is $R = 1$, transmitting 4 independent symbols over 4 time intervals.

3.1 Conditions for Full Diversity

In [17], the rank criterion was developed to evaluate the diversity achieved by space-time codes. Though introduced in the context of differential unitary space-time coding, a good measure of the quality of a square space-time code is the diversity product [18], given by

$$\zeta_v = \frac{1}{2} \min_{\mathbf{s} \neq \tilde{\mathbf{s}} \in \mathcal{V}} \left| \det(\mathbf{S} - \tilde{\mathbf{S}}) \right|^{\frac{1}{N_t}}, \quad (4)$$

where V is the set of all valid code matrices. ζ_v is bounded by $0 \leq \zeta_v \leq \sqrt{N_t E}$, and any code with $\zeta_v > 0$ achieves full diversity (from the rank criterion). Typically, larger ζ_v indicates a better code (from the determinant criterion).

The diversity product in (4) may be used as a test to search for symbol constellations that allow the modified code to achieve full diversity. To compute ζ_v for the new code, we use a property of determinants and find

$$\det(\mathbf{S} - \tilde{\mathbf{S}}) = 4 \det(\mathbf{A} - \tilde{\mathbf{A}}) \det(\mathbf{B} - \tilde{\mathbf{B}}), \quad (5)$$

where \mathbf{S} and $\tilde{\mathbf{S}}$ are matrices of the form (2) and \mathbf{A} , $\tilde{\mathbf{A}}$, \mathbf{B} , and $\tilde{\mathbf{B}}$ are Alamouti blocks defined accordingly. Furthermore,

$$\begin{aligned} \det(\mathbf{A} - \tilde{\mathbf{A}}) &= |s_1 - \tilde{s}_1|^2 + |s_2 - \tilde{s}_2|^2 \\ &= \frac{1}{2} \left[\left| d_1 + d_2 - \tilde{d}_1 - \tilde{d}_2 \right|^2 + \left| d_3 + d_4 - \tilde{d}_3 - \tilde{d}_4 \right|^2 \right], \end{aligned} \quad (6)$$

where s_1 and s_2 are symbols in Alamouti block \mathbf{A} and definitions for base symbols d_1 , d_2 , d_3 , and d_4 follow from (3). For full diversity we require $\det(\mathbf{A} - \tilde{\mathbf{A}}) \neq 0$ and $\det(\mathbf{B} - \tilde{\mathbf{B}}) \neq 0$ whenever any base symbol in \mathbf{S} differs from the corresponding symbol in $\tilde{\mathbf{S}}$. Thus,

$$d_1 - \tilde{d}_1 + d_2 - \tilde{d}_2 \neq 0, \quad (7)$$

$$d_3 - \tilde{d}_3 + d_4 - \tilde{d}_4 \neq 0, \quad (8)$$

where (7) must hold when $d_1 \neq \tilde{d}_1$ or $d_2 \neq \tilde{d}_2$, and (8) must hold when $d_3 \neq \tilde{d}_3$ or $d_4 \neq \tilde{d}_4$. A similar result follows from $\det(\mathbf{B} - \tilde{\mathbf{B}}) \neq 0$.

Consider constellations that satisfy (7) and (8) using phase rotations. For example, suppose each base symbol has the same constellation, except symbols d_2 and d_4 are rotated by an angle ϕ with respect to d_1 and d_3 . Then the constellations for $d_2 - \tilde{d}_2$ and $d_4 - \tilde{d}_4$ are also rotated by ϕ with respect to $d_1 - \tilde{d}_1$ and $d_3 - \tilde{d}_3$. Since there are only a discrete number of points in each $d_i - \tilde{d}_i$ constellation and an infinite range of phase rotations, clearly there are infinitely many phase shifts, ϕ , that offer full diversity. For uniform M -PSK base constellations, ζ_v is maximized when d_2 and d_4 are rotated by π/M with respect to d_1 and d_3 . In this case, (7) and (8) are satisfied and the code will achieve full diversity. QPSK base symbol constellations with d_2 and d_4 rotated

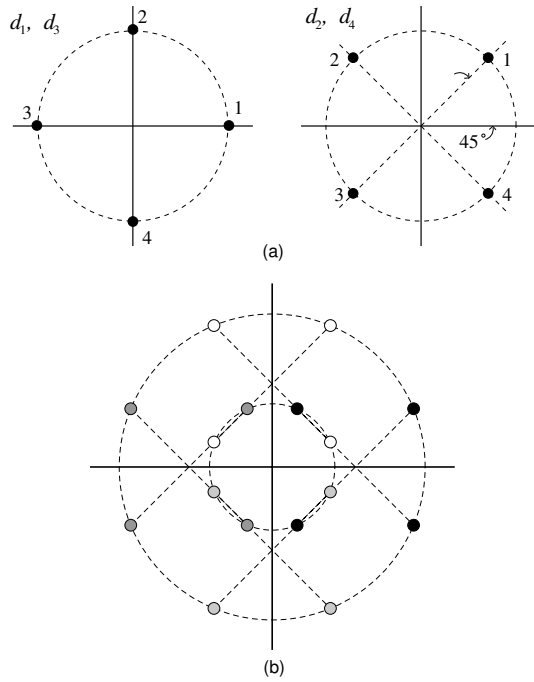


Fig. 1: (a) QPSK base symbol constellations; (b) Transmit symbol constellation.

by $\pi/4$ with respect to d_1 and d_3 are shown in Fig. 1(a). Each transmitted symbol, s_i , has 16 possible values depending on the QPSK base symbols, d_i , as depicted in Fig. 1(b).

3.2 The Optimal Receiver

When fading is known, the optimal decision rule for a space-time block code is

$$\max_{\mathbf{S}} \Re [\text{trace} (2\mathbf{R}^H \mathbf{S} \mathbf{H} - \mathbf{H}^H \mathbf{S}^H \mathbf{S} \mathbf{H})]. \quad (9)$$

The optimal receiver for the new code with $N_r = 1$ and any complex base symbols follows from (9). After simplifying, the decision rule is

$$\max_{d_1, d_2} \Re (\tilde{r}_1^* d_1 + \tilde{r}_2^* d_2 - 2z d_1 d_2^*) - \tilde{E} |d_1|^2 - \tilde{E} |d_2|^2, \quad (10)$$

$$\max_{d_3, d_4} \Re (\tilde{r}_3^* d_3 + \tilde{r}_4^* d_4 - 2z d_3 d_4^*) - \tilde{E} |d_3|^2 - \tilde{E} |d_4|^2, \quad (11)$$

where the received statistic at time i is r_i , the fading over channel j is h_j , and

$$\begin{aligned} \tilde{r}_1 &= y_1 + y_3, & \tilde{r}_2 &= y_1 - y_3, & \tilde{r}_3 &= y_2 + y_4, & \tilde{r}_4 &= y_2 - y_4, \\ y_1 &= x_1 h_1^* + x_2^* h_2, & y_2 &= x_1 h_2^* - x_2^* h_1, & y_3 &= x_3 h_3^* + x_4^* h_4, & y_4 &= x_3 h_4^* - x_4^* h_3, \\ x_1 &= r_1 + r_3, & x_2 &= r_2 + r_4, & x_3 &= r_1 - r_3, & x_4 &= r_2 - r_4, \\ z &= \frac{1}{\sqrt{2}} (|h_1|^2 + |h_2|^2 - |h_3|^2 - |h_4|^2), & \tilde{E} &= \frac{1}{\sqrt{2}} \sum_{j=1}^{N_t} |h_j|^2. \end{aligned}$$

For PSK base symbols, the optimal receiver in (10) and (11) can be simplified further since each symbol has equal energy. The resulting ML decision rule is,

$$\max_{d_1, d_2} \Re(\tilde{r}_1^* d_1 + \tilde{r}_2^* d_2 - 2z d_1 d_2^*), \quad (12)$$

$$\max_{d_3, d_4} \Re(\tilde{r}_3^* d_3 + \tilde{r}_4^* d_4 - 2z d_3 d_4^*). \quad (13)$$

Note the original STTD-OTD code is orthogonal, but the new code is not orthogonal due to the transformation in (3). The new code is quasi-orthogonal, however, since the optimal receiver decouples the base symbols into pairs, resulting in significantly reduced receiver complexity.

4 Performance

Consider the new new code with rotated QPSK symbols, having an overall spectral efficiency of 2 bits/sec/Hz. The symbol-error-rates (SER) for this and other comparable codes are presented in Fig. 2. The plot shows the new code performance against the ML performance of Papadias' improved code [12], the ABBA code [8], the rate 1/2 STTD orthogonal design [2], the constellation rotating code [14, 15], and STTD-OTD [13]. All codes shown transmit 2 bits/sec/Hz. Performance of maximal ratio combining (MRC) is also presented with SNR shifted by 6 dB ($N_t = 4$), which represents a lower bound to the achievable performance. We observe that the new code performs the same as Papadias' improved code, which outperforms all other 4 transmit antenna codes compared at significantly reduced complexity due to the partial symbol decoupling. For high SNR, the quasi-orthogonal codes have a coding gain of more than 1 dB over the next best code and perform less than 1 dB from the MRC performance bound.

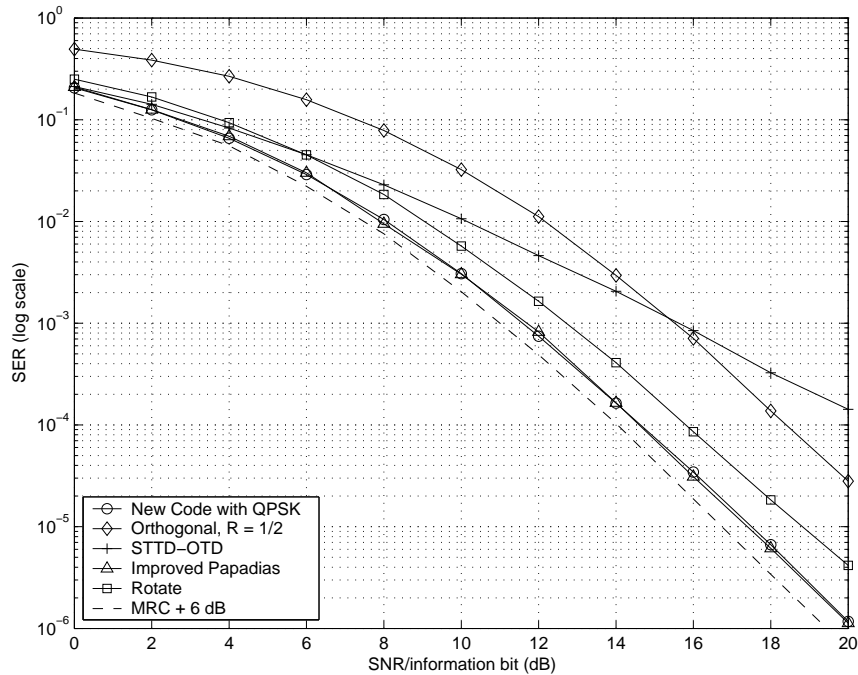


Fig. 2: Performance of the new quasi-orthogonal code with QPSK base symbols.

Performances of the new code and Papadias' improved code for various constellation rotations are shown for 12dB and 17dB in Fig. 3. The performance of these codes is practically identical, though they are different and have unique optimal rotations.

5 Conclusion

We have introduced a new, quasi-orthogonal code and have shown that it has a receiver with moderate complexity, which can decouple symbols into pairs. In general, the new code matrix does not always achieve full diversity, but with constellation phase rotations full diversity is easily attained. The new code with rotated QPSK constellations performs nearly identically to Papadias' improved quasi-orthogonal code at the same complexity. An advantage of the new code is a highly symmetrical nature for easy analysis, while a disadvantage is the expanded transmitted constellation.

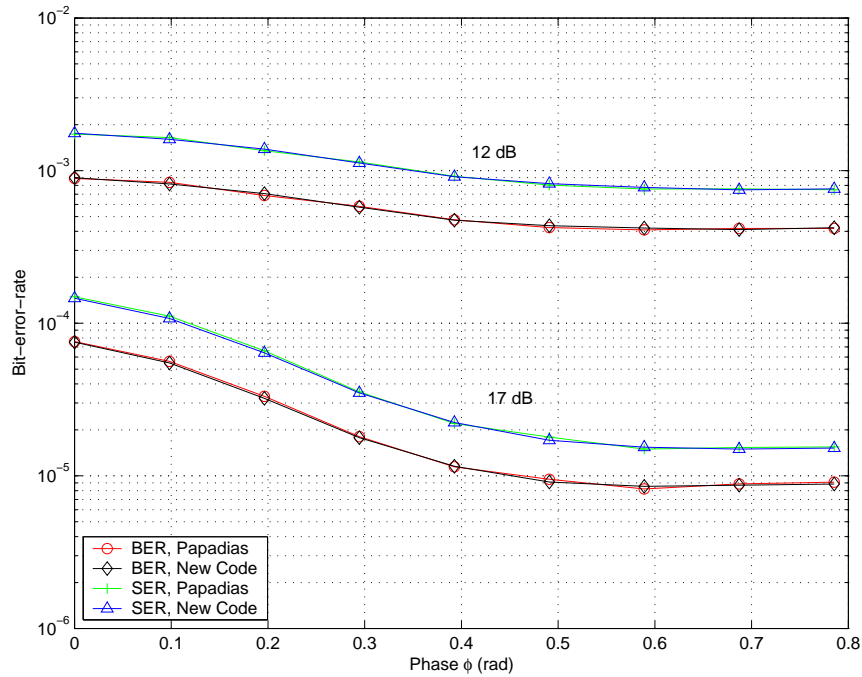


Fig. 3: Effect of rotation on quasi-orthogonal codes with QPSK at 12dB and 17dB.

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