Opportunistic Dynamic Subchannel Allocation in Multiuser OFDM Networks with Limited Feedback

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Abstract — **In this paper we present a simple scheme for subchannel allocation in OFDM multiuser networks in the presence of limited feedback, in particular, when only one bit of information per subchannel is available at the base station. Our objective is to maximize the sum rate capacity of network in the downlink transmission. We show that even with very limited feedback the sum rate capacity growth is the same as the fully informed transmission. We also extend this result to the case when subchannels are correlated.**

I. Introduction

There is ever-increasing demand for higher data rates in the next generation wireless systems. In a multiuser environment, where a large number of users share the same media for communication, efficient broadband transmission techniques that provide higher spectral efficiency have been the subject of intense research in the past few years. Orthogonal frequency devision multiplexing (OFDM) is one of the well known multicarrier techniques to combat ISI and is now an integral part of wireless standards such as 802.11a and HIPERLAN/2.

In a network of wireless users, multi-user diversity can be exploited to provide higher spectral efficiency and quality of service [1]. One way to exploit multiuser diversity gain is through opportunistic scheduling [1][2]. Previous work on opportunistic scheduling has been more focused on the frequency flat fading model [1][2][3]. However, in a network of OFDM users, only few works have utilized opportunistic schemes to increase the capacity of the system.

One of the major problems in employing an opportunistic scheme in OFDM network is the large amount of feedback required to pass to the base-station. For example in 802.11a each user has 64 subchannels and a network of 100 users requires the base station to collect 6400 real numbers from all the users. Furthermore, this information should be received error free and with no delay. To address this issue, [4] proposed an opportunistic scheme in which adjacent subchannels are clustered into groups and then only the maximum value of each cluster is fed back to the base station. But this still requires feeding back several real numbers to the base-station which may not be affordable, especially in ultra wide-band and/or fast-fading scenarios.

In this paper we propose a simple subchannel allocation scheme in which only one bit per subchannel is fed back to the base station. Thus when each user has N subchannels, only N feedback bits per user are required. We show that even this limited feedback can increase the capacity to more than twice the capacity obtained by TDMA. By introducing novel analytical techniquess, we show that the growth rate of network capacity is identical to optimal opportunistic subchannel allocation with full channel information at the base station. Furthermore we investigate the effect of correlation among channel taps to show that even when there is correlation, significant gain in terms of sum-rate capacity can be achieved by our proposed scheme.

The organization of the paper is as follows, in Section II we introduce the system model. Section III talks about the sum-rate capacity and its formulation in OFDM networks. In Section IV our proposed limited feedback dynamic subchannel allocation algorithm is discussed. The choice of the optimal threshold is addressed in Section V. Section VI contains the results about the sum-rate capacity growth in the asymptote of large number of users. In Section VII we investigate the performance of our algorithm when there is temporal correlation between each user's channel taps. Section VIII concludes the paper.

We use the following notation in this paper, $\mathbb{E}[\ ]$ refers to the expected value of a random variable, $\Re(z)$ denotes the real part of the complex number z, we use $a_n \overset{\circ}{=} b_n$ to denote the asymptotic equivalence of a_n and b_n defined as: $\lim_{n\,\to\,\infty} \frac{a_n}{b_n} = 1$. We use the natural logarithm throughout this paper so the capacity unit is in Nats/Sec/Hz.

II. System Model

For each user in the network, we consider a frequency selective linear time invariant model

$$y_{t,k} = \sum_{i=1}^{\nu} h_{i,k} x_{t-i,k} + w_{t,k} \tag{1}$$

where $x_{t,k}$ and $y_{t,k}$ are the input and the output for the k^{th} user ($k \in \{1,\ldots,K\}$) at time n respectively, w is the additive white complex Gaussian noise and uncorrelated among the users with zero mean and variance σ_w^2, $h_{t,k}$ is the n^{th} channel tap for user k and is distributed as $\mathcal{CN}(0,1)$ which is assumed to be uncorrelated among different users, although for each user, channel taps may or may not be correlated. We assume that the base-station uses OFDM for data transmission to each user. By applying cyclic prefix and IDFT, user k's channel is divided into N different sub-channels $H_{n,k}$ such that:

$$H_{n,k} = \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} h_{t,k} e^{-j\frac{2\pi n t}{N}} \tag{2}$$

We also assume that the total transmission power in the network is limited by $P_{\max}$.

III. Sum-rate Capacity

When all users share the same bandwidth, and the base-station has full information about every user's subchannels, then in order to maximize the sum-rate capacity of the network, the problem of subcarrier and power allocation to different users in the network must be solved jointly. However, this imposes a huge computational complexity at the base station. Especially if the wireless channel varies quickly, then then base-station requires an enormous computational power to rapidly compute the optimal solution for power and subchannel allocation among the users. Moreover the optimal dynamic joint power and subchannel allocation requires fast and reliable feed-forward and feedback channels for exchanging information between the users and the base station. Especially with large number of users in the network, sending an immense amount of information back and forth between the users and the base station causes a huge overhead for the network which is not economical. This motivates a low-complexity sub-optimum algorithm.

One may achieve economy of computation and communication through separation of subchannel and power allocation. It is possible to first select subchannels and then perform water-filling among all selected subchannels, but this again requires the base station to send back the optimal power allocation vector to all the users, together with the indices of their selected subchannels. Yet another suboptimal scheme is to equally allocated the total power among all subchannels and then perform the subchannel allocation among all users [5]. We adopt the latter approach in this paper.

Assuming full channel knowledge at the base station, maximizing the sum-rate capacity of the network reduces to allocating subchannel to users that have the best channel conditions. In order to avoid inter-carrier interference we allocate each frequency bin to a single user. Under this condition, maximum sum-rate capacity with equal power splitting among the subchannels is achieved when for each frequency bin we choose the user whose corresponding subchannel gain is maximum within that frequency band. The sum rate capacity in this case is given by

$$C_{full\_CSI} = \sum_{n=1}^{N} \log(1 + \mathrm{SNR} \cdot \max_k |H_{n,k}|^2) \qquad (3)$$

where $\mathrm{SNR} = \frac{P_{\max}}{N\sigma_w^2}$ is the SNR per subchannel. This subchannel selection scheme is in fact a generalization of the opportunistic scheduling in flat-fading multiuser networks [1] over N different flat fading subchannels provided by OFDM.

IV. Subchannel Allocation with Limited Feedback

The opportunistic scheme mentioned in the previous section and most of the similar subchannel allocation schemes [5] require full knowledge of the subchannel information to be available at the transmitter. However, from a practical point of view this is not affordable because a sum total of NK positive real numbers should be reliably transmitted to the base-station which is not affordable in practice . Svedman et. al [4] propose an alternative where, instead of feeding back the gain of each subchannel, each user's subchannels are divided into clusters and in each cluster the maximum value of the cluster is fed back to the base-station. This reduces the number of real values to $\frac{NK}{L}$ assuming that there are L clusters. But this still requires feeding back several real numbers to the base station without any error and delay which is still not attractive from an implementation point of view.

We propose a simple scheme where, instead of feeding back the full information of the subchannels, only one-bit of information per subchannel is fed back to the base station for subchannel allocation. For user k, the n^{th} subchannel gain $|H_{n,k}|^2$ is compared to a threshold α_n, if the subchannel gain is above the threshold a *"1"* is transmitted back to the base station otherwise a *"0"* is transmitted. So only N bits per user is required in feedback[1]. Upon receipt of all feedback bits from the users, the bases station allocates each subchannel to one of the users whose corresponding feedback bit is *"1"*. This assignment can be done via random selection or round robin scheduling among eligible users. Our claim is that by judicious choice of the threshold levels $\{\alpha_n\}$ most of the multiuser capacity gain is preserved.

Our scheduling method is distinct from that of Gesbert and Alouini [6, 7] proposed for flat fading channels in the following manner. Even though the idea of thresholding the users' channel gains has also been mentioned by Gesbert and Alouini, the requirements for their scheduling scheme are considerably different from ours. In particular, their method requires the users that have channel gains above a certain threshold to report those channel gains to the base station. This requires a feedback channel of variable-rate, but more importantly, a feedback channel that must still accommodate the transmission of real-valued numbers back to the base station. So even though in their scheme, fewer parameters than before are transmitted, still the rate is considerable. In comparison, we are interested in a strictly limited-rate feedback scenario.

V. Optimal Threshold

When channel taps are uncorrelated, i.e. $\mathbb{E}[h_{t,k}h^*_{s,k}] = \delta_{t-s}$, we can use the analytical framework developed in [8] for determining the optimal threshold value and the evaluation of the sum-rate capacity. We notice that under the assumption of uncorrelated channel taps, the subchannel gains $\{|H_{n,k}|^2\}$ are iid random variables with exponential distribution. Hence from Eq. 3 the ergodic sum-rate capacity with full CSI at the base station is:

$$C_{full\_CSI} = N\mathbb{E}[\log(1 + \mathrm{SNR}\ \max_k |H_{n,k}|^2)] \qquad (4)$$

For our 1-bit scheme we notice that $p = Pr[|H_{n,k}|^2 > \alpha_n] = e^{-\alpha_n}$ and since all the subchannels for different users are independent we conclude that the probability of receiving k *"1"*'s for the n^{th} subchannel obeys a binomial law, i.e.

$$p_k = \binom{K}{k} p^k (1-p)^{K-k} \qquad (5)$$

because of random selection of eligible subchannels (those above threshold) the ergodic sum-rate capacity conditioned on receiving exactly k *"1"*s for the n^{th} subchannel is

$$\overline{C}_k = \frac{1}{k}\sum_{i=1}^{k} C_i \qquad (6)$$

where $C_i = \int_0^\infty \log(1 + \mathrm{SNR}\ x)dF_i(x)$ and $F_i(x)$ is the CDF of the i^{th} highest absolute value of all channel

[1] One can also use the idea of clustering the subchannels to reduce the amount of feedback to $\frac{N}{L}$ bits per user

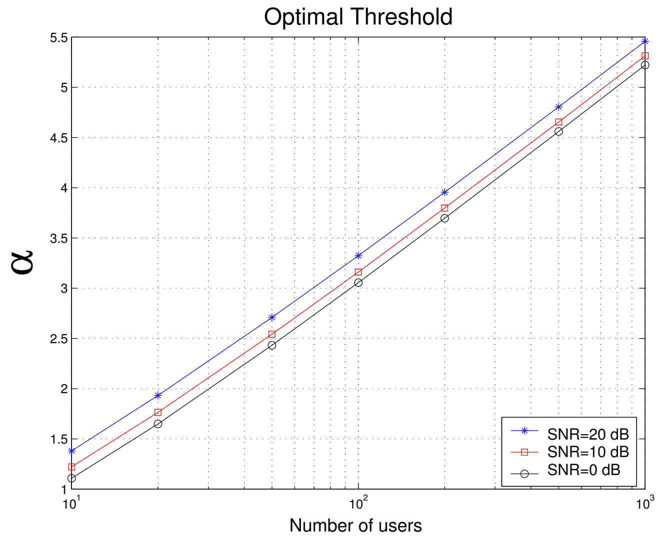

Figure 1: Optimal threshold vs. number of users for different SNR values

gains. In other words if $\{X_1, \ldots, X_K\}$ is a permutation of $\{|H_{n,1}|^2, \ldots, |H_{n,K}|^2\}$ such that $0 \le X_K \le \cdots \le X_1$, then $F_i(x) = Pr[X_i < x]$. When the channel gains are iid, it can be shown that [9]:

$$F_i(x) = \sum_{j=0}^{i-1} \binom{K}{j} (F(x))^j (1 - F(x))^{K-j} \tag{7}$$

where $F(x) = 1 - e^{-x}$ is the CDF of $|h_{n,k}|^2$ for all n k. Thus the ergodic sum-rate capacity for the case of 1-bit feedback is:

$$C_{1\_bit} = N \sum_{k=1}^{K} p_k \overline{C}_k \tag{8}$$

For the uncorrelated case, the allocation of each subchannel is unaffected by other subchannels, hence the subchannel allocation can be performed disjointly and because all subchannels have the same variance, we have $\alpha_1 = \cdots = \alpha_N = \alpha = -\log p$. In order to find the optimal α we have to solve $\frac{\partial C_{1\_bit}(p)}{\partial p} = 0$ for p. By differentiating Eq. (8) with respect to p we have

$$\sum_{k=1}^{K} (k - np) p_k \overline{C}_k = 0 \tag{9}$$

p_{opt} is the unique solution of Eq. 9 and the optimal threshold can be obtained from

$$\alpha_{opt} = -\log p_{opt} \tag{10}$$

Figure 1 depicts α_{opt} as a function of number of users K and for various values of SNR (the plot is in logarithmic scale).

VI. Asymptotic Analysis

In this section we investigate the behavior of the sum-rate capacity of OFDM networks in the asymptote of large number of users. first of all we state the following lemma which is the key tool in our asymptotic analysis.

Lemma 1 *Let $\{X_i\}_{i=1}^n$ be a sequence of positive iid random variables with finite mean μ_n and finite variance σ_n^2, also $\mathbb{E}[\log^2(X_n)] < \infty$, if $\lim_{n\to\infty} \frac{\sigma_n}{\mu_n} = 0$, then:*

$$\log(\mu_n) - \mathbb{E}[\log(X_n)] \longrightarrow 0$$

as $n \longrightarrow \infty$.

Proof: Using Tchebychev inequality for all $\epsilon > 0$ we have:

$$Pr\left[\left|\frac{X_n}{\mu_n} - 1\right| > \epsilon\right] \le \frac{\mathbb{E}[(x_n - \mu_n)^2]}{\epsilon \mu_n^2} = \frac{1}{\epsilon}\left(\frac{\sigma_n}{\mu_n}\right)^2$$

hence $\frac{X_n}{\mu_n} \xrightarrow{i.p.} 1$, now using the Slutsky's theorem [10] we have

$$\log\left(\frac{X_n}{\mu_n}\right) \xrightarrow{i.p.} 0 \tag{11}$$

On the other hand, for a random variable whose second moment is finite, we have:

$$\begin{aligned} \mathbb{E}[|Z|] &= \int_0^\infty z f_{|Z|}(z) dz \\ &= -z\left(1 - F_{|Z|}(z)\right)\Big|_{z=0}^\infty + \\ &\quad \int_0^\infty \left(1 - F_{|Z|}(z)\right) dz \end{aligned} \tag{12}$$

$\mathbb{E}[Z^2] = \int_0^\infty |z|^2 f_{|Z|}(z)dz < \infty$ therefore we should have $\lim_{z\to\infty} z^2 f_{|Z|}(z) = 0$ otherwise the integral does not converge. Using L'Hopital rule we also have:

$$\lim_{z\to\infty} z(1 - F_{|Z|}(z)) = \lim_{z\to\infty} z^2 f_{|Z|}(z) = 0$$

Thus from Eq. (12) we can conclude:

$$\mathbb{E}[|Z|] = \int_0^\infty \left(1 - F_{|Z|}(z)\right) dz = \int_0^\infty Pr[|Z| > z] dz$$

now let $Z = \log\left(\frac{X_n}{\mu_n}\right)$, then :

$$\begin{aligned} \mathbb{E}[Z^2] &= \mathbb{E}[(\log X_n - \log \mu_n)^2] \\ &\le \mathbb{E}[(\log X_n)^2] + (\log \mu_n)^2 < \infty \end{aligned} \tag{13}$$

therefore:

$$\begin{aligned} |\mathbb{E}[\log X_n] - \log \mu_n| &\le \mathbb{E}\left[\left|\log\left(\frac{X_n}{\mu_n}\right)\right|\right] \\ &= \int_0^\infty Pr\left[\left|\log\left(\frac{X_n}{\mu_n}\right)\right| > a\right] da \end{aligned} \tag{14}$$

but Eq. (11) says that for all $a > 0$, $Pr[|\log(\frac{X_n}{\mu_n})| > a] \to 0$ as $n \to \infty$. We know, $Pr[|\log(\frac{X_n}{\mu_n})| > a] \le 1$, hence using the *dominated convergence theorem* we can conclude that $\mathbb{E}[\log(X_n)] - \log(\mu_n) \to 0$, Q.E.D.

Lemma 1 states that if the probability measure associated with the random variable X_n is well concentrated around its mean value for large n, then Jensen's inequality for log() is asymptotically tight. Note that X_n can be either a *discrete* or a *continuous* random variable.

When all subchannels' gains are fully known at the base station, for each subcarrier, the base station only transmits to the users with the best corresponding subchannel. Hence the ergodic sum-rate capacity (under the assumption of uncorrelated channel taps) can be calculated from Eq. (4) by the following formula:

$$\begin{aligned} C_{full\_CSI} &= NC_1 = N\int_0^\infty \log(1+\text{SNR } x)dF_1 \\ &= NK\int_0^\infty \log(1+\text{SNR } x)e^{-x}(1-e^{-x})^{K-1}dx \end{aligned}$$

Let $\mu_1 = \int_0^\infty x dF_1$ and $\sigma_1^2 = \int_0^\infty (x-\mu_1)^2 dF_1$, then it is known [9] that:

$$\mu_1 = \sum_{i=1}^{K} \frac{1}{i} \quad \text{and} \quad \sigma_1^2 = \sum_{i=0}^{K} \frac{1}{i^2} \ ,$$

therefore $\frac{\sigma_1}{\mu_1} \to 0$ as $K \to \infty$. Combined with Lemma 1, it follows that:

$$\begin{aligned} C_{full\_CSI} &\overset{\circ}{=} N\log(1+\text{SNR } \mu_1) \\ &\overset{\circ}{=} N\log(\log K) + N\log(\text{SNR}). \end{aligned} \tag{15}$$

where $\overset{\circ}{=}$ indicates asymptotic equivalence, as defined earlier. We are interested in investigating the behavior of the sum-rate capacity of the 1-bit feedback scheduling proposed in Section IV in the asymptote of large number of users. This is accomplished via the following result.

Theorem 1 *The sum-rate capacity of a wireless network with 1-bit feedback and optimal choice of threshold, behaves as* $O(N\log(\log K) + N\log(SNR))$*, exactly the same as the sum-rate capacity of a fully informed network.*

Sketch of Proof: We start from Eq. 9 and after some algebra we arrive at:

$$C_{1\_bit} = N\sum_{i=1}^{K} \pi_i C_i \tag{16}$$

where $\pi_i = \frac{1}{Kp}\sum_{k=i}^{K} p_k,\ i = 1,\ldots,K$. Note that π_i is a valid p.m.f. because $\sum_{i=1}^{K} \pi_i = 1$, hence:

$$\begin{aligned} C_{1\_bit} &= N\sum_{i=1}^{K} \pi_i C_i \\ &= N\sum_{i=1}^{K} \pi_i \int_0^\infty \log(1+\text{SNR } x)dF_i \\ &= N\int_0^\infty \log(1+\text{SNR } x)d\left(\sum_{i=1}^{K} \pi_i F_i\right) \\ &= N\int_0^\infty \log(1+\text{SNR } x)dF_\pi \end{aligned} \tag{17}$$

where $F_\pi = \sum_{i=1}^{K} \pi_i F_i$ is a mixture probability measure of all order statistics of the exponential family. Now we show that F_π satisfies the required condition for Lemma 1.

$$\mu_\pi = \sum_{i=1}^{K} \pi_i \mu_i \tag{18}$$

where $\mu_i = \int_0^\infty x dF_i(x)$ is the mean of the i^{th} order statistics of the exponential family. In [8] it is proved that

$$\mu_\pi \overset{\circ}{=} \log K$$

and

$$\frac{\sigma_\pi}{\mu_\pi} \to 0 \quad \text{as} \quad K \to \infty \ .$$

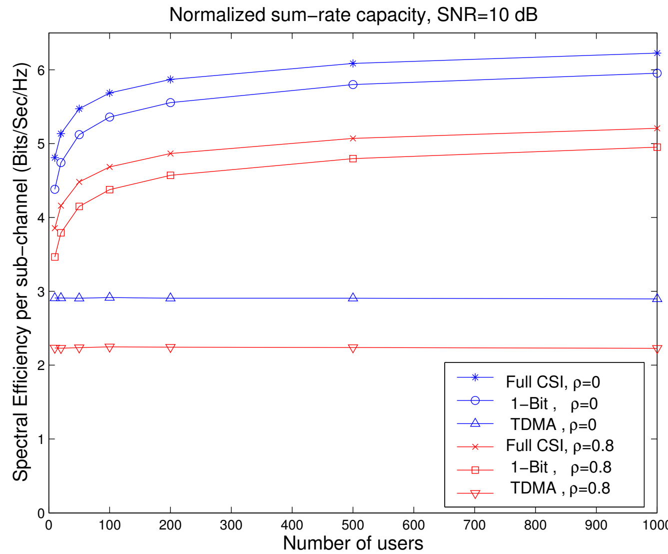

Figure 2: Sum-rate capacity (normalized by N) vs. number of users

Also from Eq. (7) it can be seen that CDF of all order statistics of the exponential family can be explained as sum of exponentials hence $F_\pi(x)$ is also consisted of weighted sum of exponential functions therefore its second order logarithmic moment exits, i.e. $\int_0^\infty (\log x)^2 dF_\pi < \infty$. Now we can apply Lemma 1 to show that:

$$\begin{aligned} C_{1\_bit} &= N\int_0^\infty \log(1+\text{SNR } x)dF_\pi \\ &\overset{\circ}{=} N\log(1+\text{SNR } \mu_\pi) \\ &\overset{\circ}{=} N\log(\log K) + N\log\text{SNR} \end{aligned} \tag{19}$$

which is exactly the same as Eq. (15)

VII. Subchannel Correlation

In this Section we assume for each user that the channel taps are correlated, but there is no dependence between different users' channels. The correlation model that we consider is an exponential decaying model described by

$$\mathbb{E}[h_{t,k}h_{s,k}^*] = \rho^{|t-s|} \tag{20}$$

This model well describes the correlation caused by a pulse-shaping filter that exits in many pratical communications standards such as GSM.

Let $\eta_{n,k}$ be the power of the n^{th} subchannel of the k^{th} user which can be calculated as:

$$\begin{aligned} \eta_{n,k} &= \mathbb{E}[|H_{n,k}|^2] \\ &= \frac{1}{N}\mathbb{E}\left[\left(\sum_{t=0}^{N-1} h_{t,k}e^{-j\frac{2\pi tn}{N}}\right)\left(\sum_{s=0}^{N-1} h_{s,k}e^{-j\frac{2\pi sn}{N}}\right)^*\right] \\ &= \frac{1}{N}\mathbb{E}\left[\sum_{t=0}^{N-1}\sum_{s=0}^{N-1} h_{t,k}h_{s,k}^* e^{-j\frac{2\pi(t-s)n}{N}}\right] \\ &= \frac{1}{N}\sum_{t=0}^{N-1}\sum_{s=0}^{N-1} \rho^{|t-s|}e^{-j\frac{2\pi n}{N}(t-s)} \end{aligned}$$

by change of summing index to $u = t - s$ we get

$$\begin{aligned}\eta_{n,k} &= \sum_{u=-(N-1)}^{N-1} (1 - \frac{|u|}{N})\rho^{|u|} e^{-j\frac{2\pi n}{N}u} \\ &= \Re\{\beta_n\} - 1 \end{aligned} \quad (21)$$

where $\beta_n = \frac{1}{\sqrt{N}} \sum_{m=1}^{N-1} b_m e^{-j\frac{2\pi n}{N}m}$ is the discrete Fourier transform (DFT) of the sequence $b_m = (\sqrt{N} - \frac{m}{\sqrt{N}})\rho^m$. Assuming $\rho^N \ll 1$, after some algebra we obtain the following expression for $\eta_{n,k}$:

$$\eta_{n,k} \approx \frac{1-\rho^2}{1+2\rho\cos\theta_n+\rho^2} + \frac{2\rho\cos\theta_n + 4\rho^2 + 2\rho^3\cos\theta_n}{N(1+2\rho\cos\theta_n+\rho^2)^2} \quad (22)$$

where $\theta_n = \frac{2\pi n}{N}$. Notice that for a given n, $\{H_{n,k}\}$'s are i.i.d. across different users, hence $\eta_{n,k}$ does not depend on k. Thus correlation between taps, leads to subchannels with different qualities. On the other hand exact calculation of the optimal threshold for this case is not mathematically tractable. So we propose a suboptimal solution for quantizing the subchannels with one bit. In fact for the n^{th} frequency bin, we divide the subchannel gains by $\eta_{n,k}$ and then compare the normalized channel gain by the optimal threshold calculated in Section V, if

$$\frac{|H_{n,k}|^2}{\eta_{n,k}} \geq \alpha_n$$

the feedback bit is set to "1", otherwise it is set to "0".
Fig. 2 is the simulation result based on the proposed algorithm for SNR=10 dB. As can be seen in the figure for both uncorrelated and correlated cases, the sum-rate capacity growth of our scheme is the same as the opportunistic subchannel selection with full information available at the base station. Moreover the capacity achieved by our scheme is much higher than TDMA scheduling and only slightly lower than the full CSI sum-rate capacity.

VIII. Conclusion and Future Work

In this paper we proposed a simple algorithm for dynamic subchannel allocation in multiuser OFDM networks. Our scheme only requires one bit per subchannel per user and, despite its very low feedback rate, achieves the same capacity growth as the subchannel allocation with full channel information for all users at the base station. For future work we would like to extend this limited feedback approach to MIMO system. Also another problem of interest is the impact of channel estimation error also the effect of feedback error and delay on the performance of our scheme which remains to be a topic of future research.

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