Delay-limited capacity and maximum throughput of spatially correlated multiple antenna systems under average and peak-power constraints

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Abstract — **The delay-limited capacity is defined as the transmission rate that can be guaranteed in all fading states under finite long-term power constraints. For the single-input single-output Rayleigh fading channel it is zero. In contrast it is greater than zero in multiple antenna channels but depends on the properties of the fading channel, e.g. on the spatial correlation. In this work, we prove that the delay-limited capacity is Schur-concave with respect to the spatial correlation. In addition to the average power constraint, we apply a peak-power constraint which limits the kurtosis of the input signal. We derive the delay-limited capacity for this general class of multiple antenna channels with correlation under peak-power and long-term power constraint.**

Without the stringent delay constraint, the maximum throughput is defined as the transmission rate times successful transmission probability. When the transmitter is uninformed, the maximum throughput is achieved for small SNR by using only one transmit antenna and for high SNR by using all available transmit antennas. When the transmitter has perfect channel knowledge, the optimal power allocation under long-term power constraint is analyzed and the impact of correlation is discussed by numerical simulations.

I. Introduction

Multiple antenna systems were extensively studied in terms of their performance and achievable rates [18, 8]. Recently, the impact of correlation on the average mutual information and on the outage probability was analyzed in [13, 4]. The capacity of fading channels with channel state information (CSI) at the transmitter was studied in [10].

In this work, we consider the delay-limited capacity of quasi-static flat-fading multiple antenna channels. The delay-limited capacity, also called zero-outage capacity, is the achievable transmission rate which can be guaranteed for all fading states. In [7, p. 2633], the delay-limited capacity and the connection to compound channels are discussed. In order to obtain a delay-limited capacity greater than zero, the transmitter performs temporal power allocation under a long-term power constraint. In [12], the authors studied the delay-limited capacity region for the multiuser MAC. The delay-limited capacity for single antenna Rayleigh fading channels is zero, whereas it is greater than zero for channels with more than one degree of freedom, e.g. multiple antenna channels or multi-carrier channels. In the first part, we analyze the impact of spatial correlation on the delay-limited capacity in multiple antenna systems. Using Majorization theory, we show that the delay-limited capacity is Schur-concave with respect to the antenna correlation. In addition, we apply a peak-power constraint which limits the kurtosis of the input signal. In [14], the capacity of fading channels under average and peak-power constraints was derived and a coding theorem and its converse was proved. Here, we derive the delay-limited capacity under average and peak-power constraint for spatially correlated multiple antenna channels.

The transmission rate can be significantly increased by relaxing the stringent delay constraint. The throughput of the overall system is then given as the product of the rate times the probability of successful transmission [3]. Even without CSI at the transmitter, the maximum throughput is greater than zero. The optimal transmit strategy depends on the SNR and is to allocate equal power to a subset of the available antennas. Based on the results in [4], we show that for small SNR the maximum throughput is attained with only one transmit antenna and for high SNR all available antennas should be used. Finally, the case when the transmitter has perfect CSI is analyzed. In [6], the authors provide an algorithm which yields the optimal power allocation for outage probability minimization. We apply this algorithm for the problem at hand and discuss the impact of correlation on the maximum throughput by numerical simulations. In [2], different throughput measures for delay-constrained communications are introduced. We compare these measures and their properties to our approach.

II. Channel model and spatial correlation

A. Channel model

We consider the standard quasi-static flat-fading multiple antenna channel model. In the case in which the transmitter has multiple antennas it is given by

$$y = \boldsymbol{x}^H \boldsymbol{h} + n$$

with complex $n \times 1$ transmit vector $\boldsymbol{x}$, channel vector $\boldsymbol{h}$ $(n \times 1)$, circularly symmetric complex Gaussian noise n with variance $\frac{\sigma_n^2}{2}$ per dimension. In the case in which the receiver has multiple antennas, the received vector is given by $\boldsymbol{y} = x\boldsymbol{h} + \boldsymbol{n}$. Here, the scalar x is the transmitted signal, $\boldsymbol{h}$ is the channel vector, and $\boldsymbol{n}$ is the noise vector with independent and identically zero mean circularly symmetric complex Gaussian distributed entries with variance σ_n^2. For convenience, we define the inverse noise variance as $\rho = 1/\sigma_n^2$. We have a transmit power constraint p. Therefore, the SNR is given by $p\rho$. We assume that the receiver knows $\boldsymbol{h}$ perfectly.

Next, we describe the stochastic properties of the block-fading channel for the case when the transmitter has multiple antennas (multiple-input single-output MISO). The single-input multiple-output (SIMO) case is analogue. The covariance matrix of the channel vector realizations is given by $\boldsymbol{R} = \mathbb{E}\left(\boldsymbol{h}\boldsymbol{h}^H\right)$[1]. The eigenvalue decomposition of the channel covariance matrix $\boldsymbol{R}$ is given by $\boldsymbol{R} = \boldsymbol{U}_R \boldsymbol{M}_R \boldsymbol{U}_R^H$ with diagonal matrix $\boldsymbol{M}_R$ with eigenvalues $\mu_1, ..., \mu_n$[2]. The correlation of the channel vectors arises in the common downlink transmission scenario in which the base station is un-obstructed [16]. We follow the model in [9] in which the subspaces and directions of the paths between the transmit antennas and the receive cluster change slower than the actual attenuation of each path. The eigenvectors in $\boldsymbol{U}_R$ correspond to the dominant directions and the eigenvalues are the average power in these directions.

B. Measure of spatial correlation

In order to compare two correlation scenarios, the following framework can be applied: For two vectors $\boldsymbol{x}, \boldsymbol{y} \in R^n$ one says that the vector $\boldsymbol{x}$ majorizes the vector $\boldsymbol{y}$ and writes $\boldsymbol{x} \succ \boldsymbol{y}$ if

$$\sum_{k=1}^{m} x_k \geq \sum_{k=1}^{m} y_k \quad \forall m = 1, ..., n-1, \text{ and } \sum_{k=1}^{n} x_k = \sum_{k=1}^{n} y_k.$$

A real-valued function Φ defined on $\mathcal{A} \subset R^n$ is said to be *Schur-convex* on $\mathcal{A}$ if from $\boldsymbol{x} \succ \boldsymbol{y}$ on $\mathcal{A}$ follows $\Phi(\boldsymbol{x}) \geq \Phi(\boldsymbol{y})$. Similarly, Φ is said to be *Schur-concave* on $\mathcal{A}$ if from $\boldsymbol{x} \succ \boldsymbol{y}$ on $\mathcal{A}$ follows $\Phi(\boldsymbol{x}) \leq \Phi(\boldsymbol{y})$.

For further information about majorization theory see [15]. The following definition provides a measure for comparison of two covariance matrices. The channel covariance matrix $\boldsymbol{R}^1$ is more correlated than $\boldsymbol{R}^2$ if and only if $\sum_{l=1}^{m} \mu_l^1 \geq \sum_{l=1}^{m} \mu_l^2$ for $m = 1...n-1$ and $\sum_{l=1}^{n} \mu_1^1 = \sum_{l=1}^{n} \mu_1^2$.

It can be shown that vectors with more than two components cannot be totally ordered. This is a problem of all possible orders for comparing correlation vectors. The case in which the transmit antennas are fully correlated corresponds to $\mu_1 = n$, $\mu_2 = ... = \mu_n = 0$. The case in which the transmit antennas are fully uncorrelated corresponds to $\mu_1 = \mu_2 = ... = \mu_n = 1$.

We need the following result (see [15, Theorem 3.A.4]) which is sometimes called Schur's condition. It provides an approach for testing whether some vector valued function is Schur-convex or not. Schur's condition for the Schur-convexity of a symmetric function $f(\boldsymbol{x})$ is given as [15, p. 57]

$$(x_1 - x_2)\left(\frac{\partial f}{\partial x_1} - \frac{\partial f}{\partial x_2}\right) \geq 0 \tag{1}$$

for all $x_1, x_2 \in \mathcal{I}^n$. Furthermore, $f(\boldsymbol{x})$ is a Schur-concave function on $\mathcal{I}^n$ if $f(\boldsymbol{x})$ is symmetric and

$$(x_1 - x_2)\left(\frac{\partial f}{\partial x_1} - \frac{\partial f}{\partial x_2}\right) \leq 0 \tag{2}$$

for all $x_1, x_2 \in \mathcal{I}^n$.

III. Analysis of delay-limited capacity

We follow the definition of the delay-limited capacity in [6]. The instantaneous mutual information of the multiple antenna channel is as function of the SNR which depends on ρ and the instantaneous transmit power p, and the channel realization $\boldsymbol{h}$

$$I(\rho, p, \boldsymbol{h}) = \log\left(1 + \rho p ||\boldsymbol{h}||^2\right). \tag{3}$$

A. Long-term power constraint

The capacity vs. outage probability ϵ - or ϵ-capacity under long-term power constraint

$$\mathbb{E}\left[p(\boldsymbol{h})\right] \leq P \tag{4}$$

is a function of the SNR ρ, the long-term power constraint P, and the channel correlation $\boldsymbol{\mu} = [\mu_1, ..., \mu_n]$ and it is the rate R

$$\arg\max R \text{ s.t. } Pr\left[I(\rho, p, \boldsymbol{h}) \leq R\right] \leq \epsilon. \tag{5}$$

The delay-limited capacity is defined as the 0-capacity, i.e. for all fading states the outage probability is zero with probability one. In order to achieve this with minimal power, the optimal power allocation $p^*(\boldsymbol{h})$ has to fulfill

$$p^*(\boldsymbol{h}) = \frac{2^R - 1}{\rho ||\boldsymbol{h}||^2} \tag{6}$$

for all fading states $\boldsymbol{h}$. The rate R has to be chosen such that $\mathbb{E}\left[p^*(\boldsymbol{h})\right] = P$. The average of p^* in (6) is given by

$$\mathbb{E}\left[p^*(\boldsymbol{h})\right] = \frac{2^R - 1}{\rho} \mathbb{E}\left[\frac{1}{||\boldsymbol{h}||^2}\right].$$

In order to fulfill the power constraint, p^* is given by

$$p^*(\boldsymbol{h}) = \frac{P}{\mathbb{E}\left[\frac{1}{||\boldsymbol{h}||^2}\right]} \frac{1}{||\boldsymbol{h}||^2}$$

and the delay-limited capacity is

$$C^d(\rho, P, \boldsymbol{\mu}) = \log_2\left(1 + \rho \frac{P}{\mathbb{E}\left[\frac{1}{||\boldsymbol{h}||^2}\right]}\right). \tag{7}$$

Note that the delay-limited capacity in (7) depends on the channel correlation by the average of the inverse channel norm. In order to quantify the effect of correlation on the delay-limited capacity, we prove the following theorem.

[1] For SIMO, the covariance matrix is given by $\boldsymbol{R} = \mathbb{E}\left(\boldsymbol{h}^H \boldsymbol{h}\right)$.

[2] We assume without loss of generality, that the channel covariance matrix eigenvalues are decreasingly ordered, i.e. $\mu_1 \geq \mu_2 \geq ... \geq \mu_n \geq 0$.

Theorem III.1 *The delay-limited capacity is a Schur-concave function with respect to the vector of the eigenvalues of the spatial correlation matrix, i.e.*

$$\boldsymbol{\mu}_1 \succeq \boldsymbol{\mu}_2 \Longrightarrow C^d(\rho, P, \boldsymbol{\mu}_1) \leq C^d(\rho, P, \boldsymbol{\mu}_2).$$

Proof We study the vector-valued function

$$f(\boldsymbol{\mu}) = \mathbb{E}\frac{1}{||\boldsymbol{h}||^2} = \mathbb{E}\left[\frac{1}{\sum_{k=1}^n \mu_k w_k}\right]$$

with independent random standard exponential distributed w_k. The first derivative of f with respect to μ_1 and μ_2 is given by

$$\frac{\partial f(\boldsymbol{\mu})}{\partial \mu_1} = -\mathbb{E}\left[\frac{w_1}{\left(\sum_{k=1}^n \mu_k w_k\right)^2}\right] \quad (8)$$

$$\frac{\partial f(\boldsymbol{\mu})}{\partial \mu_2} = -\mathbb{E}\left[\frac{w_2}{\left(\sum_{k=1}^n \mu_k w_k\right)^2}\right]. \quad (9)$$

The difference Δ between the two derivatives in (8) and (9) is given by

$$\begin{aligned}\Delta &= \mathbb{E}\left[\frac{w_2 - w_1}{\left(\sum_{k=1}^n \mu_k w_k\right)^2}\right] \\ &= \int_0^\infty \prod_{k=3}^n \frac{1}{1+t\mu_k}\Big(\frac{1}{1+t\mu_1}\frac{t}{(1+t\mu_2)^2} \\ &\quad -\frac{1}{1+t\mu_2}\frac{t}{(1+t\mu_1)^2}\Big)dt \\ &= \int_0^\infty \prod_{k=3}^n \frac{1}{1+t\mu_k}\frac{t^2}{(1+t\mu_1)^2(1+t\mu_2)^2}\left(\mu_1 - \mu_2\right)dt \\ &\geq 0 \quad (10)\end{aligned}$$

The first identity in (10) follows from the fact that

$$\mathbb{E}\left[\frac{1}{\sum_{k=1}^n \mu_k w_k}\right] = \int_0^\infty \prod_{k=1}^n \frac{1}{1+t\mu_k}dt.$$

Since (10) shows that Schur's condition in (1) is fulfilled for all $\mu_1 \geq \mu_2$ it follows that $\frac{1}{\mathbb{E}\left[\frac{1}{||\boldsymbol{h}||^2}\right]}$ is Schur-concave and the proof is completed. ∎

Remark The completely correlated multiple antenna channel has zero delay-limited capacity, completely uncorrelated multiple antenna channel has the delay-limited capacity

$$C^d(\rho, P, \mathbf{1}) = \log_2\left(1 + \rho P\frac{n-1}{n}\right) \quad (11)$$

which converges for n approaching infinity to the capacity of the AWGN channel $\log_2(1+\rho P)$. Furthermore, for the case when all μ_k are different, i.e. $\mu_k \neq \mu_j$ for all $k \neq j$, we have the following closed form expression for the expectation

$$\mathbb{E}\left[\frac{1}{\sum_{k=1}^n \mu_k w_k}\right] = \sum_{k=1}^n \log(\mu_k)\prod_{j\neq k}\left(\frac{1}{\mu_k - \mu_j}\right). \quad (12)$$

Using (12), the delay-limited capacity can be easily computed in closed form. Note that the loss due to correlation between completely uncorrelated and fully correlated antennas grows with the SNR and with the number of antennas. For the completely correlated case, the delay-limited capacity is zero.

B. Peak power constraint

The delay-limited capacity was analyzed under the usual long-term power constraint in (4). However, in many practical systems, there is a limitation on the peak power due to the non-linearity of the amplifiers, and also for compliance with other existing standards. Therefore, we apply a peak-power constraint by restricting the Kurtosis of the input distribution, see e.g. [11, Section 3.1], i.e.

$$\mathbb{E}\left[p(\boldsymbol{h})^2\right] \leq \kappa \mathbb{E}\left[p(\boldsymbol{h})\right]^2. \quad (13)$$

The constraint in (13) limits the Kurtosis which is a measure of the peakedness of the input signal to be less than or equal to $\kappa < \infty$.

Remark The kurtosis of the power distribution $p(\boldsymbol{h})$ depends only on the channel statistics, i.e.

$$\kappa = \frac{\mathbb{E}\left[\frac{1}{||\boldsymbol{h}||^4}\right]}{\mathbb{E}\left[\frac{1}{||\boldsymbol{h}||^2}\right]^2}. \quad (14)$$

Lemma III.2 *The delay-limited capacity for the multiple-antenna correlated flat Rayleigh fading channel under long-term power constraint (4) and peak power constraint (13) is greater than zero only if rank*$(\boldsymbol{R}) > 2$.

Proof Let the eigenvalues of the correlation matrix $\boldsymbol{R}$ be fixed in decreasing order, i.e. $\mu_1 \geq \mu_2 \geq ... \geq \mu_r > \mu_{r+1} = ... = \mu_n = 0$. r denotes the rank of $\boldsymbol{R}$. The fourth moment can be upper and lower bounded by

$$\frac{1}{\mu_1^2}\mathbb{E}\left[\frac{1}{v^2}\right] \leq \mathbb{E}\left[\frac{1}{(\sum_{k=1}^n \mu_k w_k)^2}\right] \leq \frac{1}{\mu_r^2}\mathbb{E}\left[\frac{1}{v^2}\right] \quad (15)$$

with random variable $v = \sum_{k=1}^r w_k$ standard χ^2- distributed with $2r$ degrees of freedom. Therefore, the fourth moment is finite if and only if $\mathbb{E}\left[\frac{1}{v^2}\right] < \infty$. The expectation is given by[3]

$$\mathbb{E}\left[\frac{1}{v^2}\right] = \frac{1}{(r-2)(r-1)}. \quad (16)$$

Since the expectation in (16) is infinity for $r = 1$ and $r = 2$, the statement of Lemma III.2 follows. The kurtosis is given by $\kappa = \frac{1}{r-2}$. ∎

[3]In general, the $k-th$ moment of $\frac{1}{v}$ is given by $\frac{\Gamma(r-k)}{\Gamma(r)}$.

Concluding the delay-limited capacity analysis for multiple antenna systems, there are three possible cases for the delay-limited capacity under long-term power constraint and peak-power constraint: Either the delay-limited capacity is zero, because the rank of the correlation matrix $\boldsymbol{R}$ is less than three; or the delay-limited capacity is limited by the long-term power constraint and it is given by (7) with (12); or the peak-power constraint is active. This leads to the following Theorem III.3. Denote the second moment as $m_2 = \mathbb{E}\left[\frac{1}{\sum_{k=1}^n \mu_k w_k}\right]$ in (12) and the fourth moment[4] as $m_4 = \mathbb{E}\left[\frac{1}{(\sum_{k=1}^n \mu_k w_k)^2}\right]$.

Theorem III.3 *The delay-limited capacity of the Rayleigh flat-fading multiple antenna channel with n antennas and correlation $\boldsymbol{R}$ with rank r and eigenvalues $\mu_1, ..., \mu_n$ under long-term power constraint $m_2 \leq P$ and peak-power constraint $m_4 \leq \kappa m_2$ is given by*

$$C^d(\rho, P, \kappa) = \begin{cases} 0 & \text{if } r \leq 2 \text{ or } \frac{m_4}{m_2^2} > \kappa \\ \log_2(1 + \frac{\rho P}{m_2}) & \text{if } r > 2 \text{ and } \frac{m_4}{m_2^2} \leq \kappa \end{cases}$$

Note, that the fourth moment m_4 is Schur-concave with respect to $\boldsymbol{\mu}$, too. This can be shown by following the proof of Theorem III.1.

IV. Maximum throughput analysis

In this section, we relax the stringent delay requirements from the last section and allow for arbitrary delay. The delay can be exploited either by increasing the length of one codeword or by introducing some kind of automatic repeat request (ARQ). If the block length of the codeword is increased, the outage probability for this codeword is reduced. Here, following [2] , we focus on the "Maximum Zero-Outage Throughput". The receiver requests a retransmission as long as outages occur until the codeword is sucessfully decoded. Therefore, the complete information is reliable transmitted. The probability that a codeword has to be transmitted s times is given by

$$Pr\Big[\text{codeword } s\text{-times transmitted}\Big] = Pr\left[\bigcap_{i=1}^{s-1} out_i\right]\left[1 - Pr\left(out_s \middle| \bigcap_{i=1}^{s-1} out_i\right)\right] \quad (17)$$

where out_s means an outage in retransmission s. If the receiver considers only the actual packet for decision, the probability of s times transmission is given by

$$\prod_{k=1}^{s-1} Pr\Big[k\text{-th outage}\Big] \cdot \Big(1 - Pr\Big[s\text{-th outage}\Big]\Big) \quad (18)$$

under the assumption of an independent block-fading channel. The maximum throughput is then defined to be

$$T(\rho, P) = \sup_R \frac{R}{\mathbb{E}[S]}$$

with the average service time $\mathbb{E}[S]$. Using (18), the maximum throughput for this simple retransmission scheme is given by

$$T^{MZT}(\rho, P) = \max_{R \geq 0} R\left(1 - Pr\left[I(\rho, P, h) \leq R\right]\right). \quad (19)$$

In [2] the quantity in (19) is called "Maximum Zero-Outage Throughput". Here, a transmission of a certain amount of data is not guaranteed within a limited delay. The measure in (19) is contrary to the delay-limited capacity in the last section. There are channel realizations in which more bits are reliable transmitted than $T^{MZT}(\rho, P)$ and there are realizations in which less bits are transmitted. The probability that $T^{MZT}(\rho, P)$ bits (out of R bits) are transmitted without errors is given by $Pr\left[I(\rho, P, h) \leq T^{MZT}(\rho, P)\right]$.

The connection between the delay-limited capacity and the maximum throughput becomes clear for rates that are smaller than the delay-limited capacity. In this case, the optimal power allocation which minimizes the outage probability corresponds to the optimal power allocation in the delay-limited case. Therefore, the achievable rates are equal, too.

A. Uninformed transmitter

When the transmitter is uninformed, the optimal temporal power allocation is equal power allocation. In the SIMO case with uncorrelated receive antennas, we obtain

$$\begin{aligned} f(R) &= R\left(1 - Pr\left[I(\rho, P, \boldsymbol{h}) \leq R\right]\right) \\ &= Re^{-n\frac{2^R-1}{\rho}}\left(\sum_{i=0}^{n-1} \frac{\left(n\frac{2^R-1}{\rho}\right)^i}{\Gamma(i+1)}\right). \end{aligned} \quad (20)$$

If the receive antennas are correlated, the maximum throughput additionally depends on the eigenvalues $\boldsymbol{\mu}$

$$\begin{aligned} T^{MZT}(\rho, P) &= \max_{R \geq 0} R\left(1 - Pr\left[\sum_{k=1}^n \mu_k w_k \leq \frac{2^R - 1}{\rho P}\right]\right) \\ &= \max_{R \geq 0} R \sum_{k=1}^n \prod_{l \neq k} \frac{1}{1 - \frac{\mu_l}{\mu_k}}\left(1 - e^{-\frac{2^R-1}{\rho P \mu_k}}\right) \end{aligned} \quad (21)$$

for completely disjunct eigenvalues, i.e. $\mu_k \neq \mu_l$ for all $k \neq l$. From the outage probability analysis in [4] we know that the outage probability is for small SNR values (or high rates) Schur-concave and for high SNR values (or low rates) Schur-convex. This behavior carries over to the maximum throughput in (21).

B. Informed transmitter

The optimal power allocation for minimization of the outage probability with informed transmitter is characterized in [6]. The optimization problem

$$\min Pr\left[I(\rho, p(\boldsymbol{h}, \boldsymbol{h}) \neq R\right] \quad \text{s.t. } \mathbb{E}p(\boldsymbol{h}) \leq P \quad (22)$$

is solved by the power allocation

$$p^*(\boldsymbol{h}) = \begin{cases} \frac{2^R-1}{\rho||\boldsymbol{h}||^2} & \text{if } \frac{2^R-1}{\rho||\boldsymbol{h}||^2} < s^* \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

[4]Unfortunately, there is no simple closed form expression for the fourth moment m_4 for correlated channels.

with $s^* = \sup\{s : \int_{\frac{2^R-1}{\rho s}}^{\infty} \frac{2^R-1}{\rho\alpha} pdf(\alpha) d\alpha < P\}$. Note that for s approaching infinity, the optimal power allocation converges to the optimal power allocation in the delay-limited capacity case.

In the case in which the antennas are uncorrelated, the pdf of $\alpha = ||\boldsymbol{h}||^2$ is given by $\frac{\alpha^{n-1} exp(-\alpha)}{\Gamma(n)}$ and the optimal s^* solves

$$\frac{P\rho}{2^R - 1} = \frac{\Gamma(n-1, \frac{2^R-1}{\rho P s^*})}{\Gamma(n)} \tag{24}$$

with the incomplete Gamma-function [1, 6.5.3] $\Gamma(a, z) = \int_z^{\infty} \exp(-t) t^{a-1} dt$.

In the case where the antennas are correlated and all μ_k are disjunct, the pdf of α is given as

$$pdf(\alpha) = \sum_{k=1}^{n} \prod_{l \neq k} (1 - \frac{\mu_l}{\mu_k})^{-1} e^{-\frac{\alpha}{\mu_k}}$$

and the optimal s^* solves

$$P = \sum_{k=1}^{n} \prod_{l \neq k} (1 - \frac{\mu_l}{\mu_k})^{-1} e^{-\frac{2^R-1}{\rho P s^* \mu_k}}. \tag{25}$$

Using the representation of s^* in (24) and (25), respectively, the optimal temporal power allocation can be numerically computed using (23).

C. Incremental diversity

The maximum throughput in (19) was computed for the simplest retransmission scheme, in which the receiver decides each codeword independently of formerly received codewords. We assume equal power allocation in the following. If the receiver combines all replicas of received codewords optimally (maximal ratio receive combining MRRC), the outage probability in the s-th retransmission is given by

$$\begin{aligned} Pr\left[out_s\right] &= Pr\left[\sum_{l=1}^{s}\sum_{m=1}^{n} \mu_m w_{lm} \leq \frac{2^R-1}{\rho P}\right] \\ &= Pr\left[\sum_{k=1}^{n} \mu_k v_k \leq \frac{2^R-1}{\rho P}\right] \end{aligned} \tag{26}$$

with v_k as a χ^2_{2s} independent distributed random variable for $1 \leq k \leq n$. In [5], it was shown that the probability of the sum of weighted Gamma distributed random variables $Pr\left[\sum_{k=1}^{n} \mu_k v_k \leq t\right]$ is Schur-convex in $\boldsymbol{\mu} = [\mu_1, ..., \mu_n]$ if

$$\{\boldsymbol{\mu} : \min_{1 \leq i \leq n} \mu_i \geq \frac{t}{ns+1}\}$$

and it is Schur-concave if

$$t \geq (ns+1) \sum_{k=1}^{n} \mu_k.$$

Using Bayes rule, we obtain for the probability for s-th retransmissions the difference between the probability of an outage in $s-1$ transmissions and the probability of an outage in s transmissions which is for the completely uncorrelated channel with $\beta = \frac{2^R-1}{\rho P}$ given by

$$Pr\left[\text{codeword } s\text{-times transmitted}\right] = \frac{\exp(-\beta)\beta^{s-1}}{\Gamma(s)}.$$

The "Maximum Zero-Outage Throughput" with incremental diversity reads then

$$T^{IR}(\rho, P) = \sup_R \frac{R\rho P}{2^R + \rho P - 1}.$$

V. Illustrations

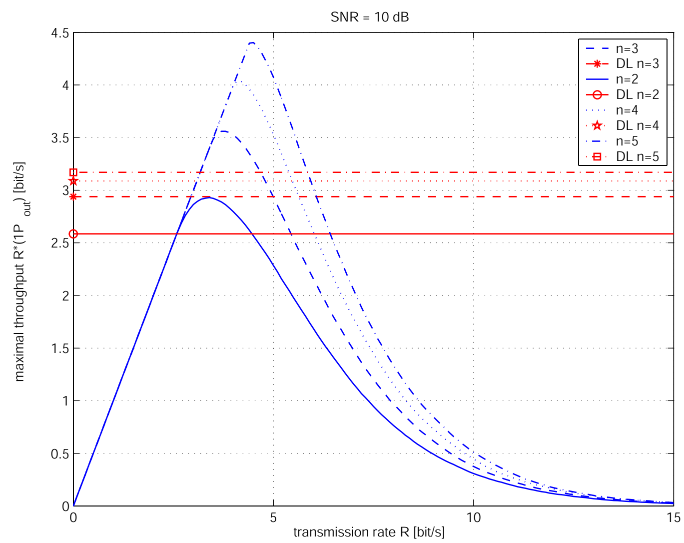

Figure 1: Informed transmitter: successful transmission rate over transmission rate for different numbers of receive antennas in comparison to delay-limited capacity (horizontal) at SNR 10 dB.

In figure 1, the successful transmission rate $T(\rho, P)$ is shown over transmission rate R. It can be observed that the delay-limited capacity $C^d(\rho, P)$ is always smaller than the maximum throughput $T(\rho, P)$ because of the delay constraint. For small rates up to C^d there does not occur any outage and the curve is the bisecting line. Then there is a rate range in which outages occur but the resulting throughput is higher than the delay-limited capacity. Finally, the outages dominate and the throughput descends to zero as the outage probability tends to one for higher rates R.

In figure 1, we observe that the more receive antennas are used, the higher is the delay-limited capacity and the higher is the maximum throughput for a SNR of 10 dB.

In figure 2, it can be observed that the impact of correlation on the maximum throughput is different for small and high SNR values. For small SNR values, the maximum throughput is increased with correlation while for high SNR values, the maximum throughput decreases with increasing correlation.

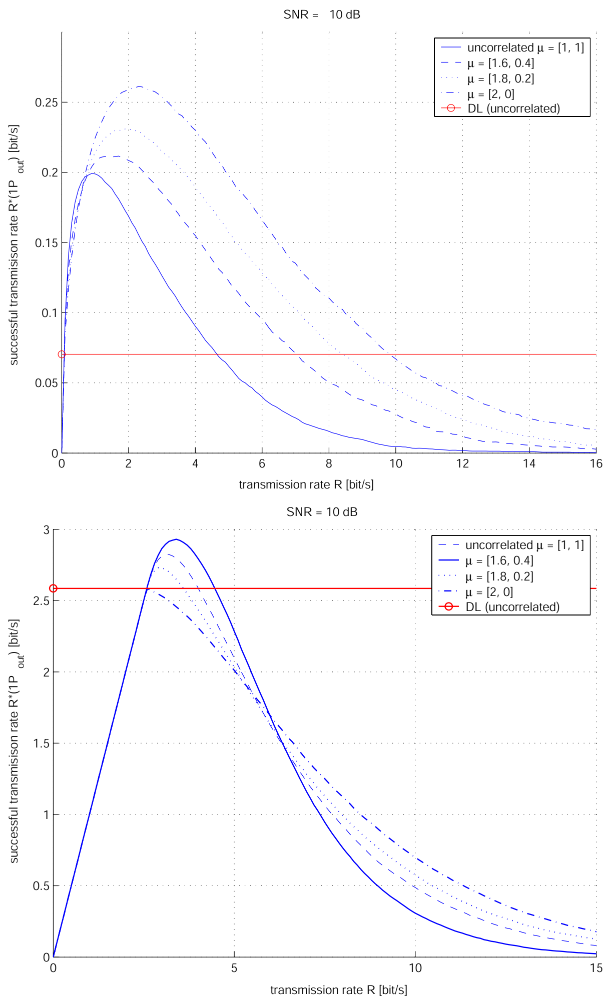

Figure 2: Informed transmitter: maximum throughput over transmission rate for spatially correlated receive antennas at SNR -10 and 10 dB.

VI. Conclusions and Future Work

In this work, we studied first the delay-limited capacity of spatially correlated multiple antenna systems under long-term power constraints and additionally under peak-power constraint. The delay-limited capacity is Schur-concave with respect to the correlation properties of the channel, i.e. it is highest for completely uncorrelated antennas. In addition to the delay limited capacity, the maximum throughput was analyzed which is defined as the maximum of the rate times successful transmission probability. The uninformed transmitter and the perfectly informed transmitter were studied. The maximum throughput has no clear behavior with respect to the channel correlation. The optimal power allocation was derived. The properties of the maximum throughput compared to the delay-limited capacity were illustrated by numerical simulations.

The kurtosis depends only on the multiple antenna channel statistics. Currently, the impact of correlation on the kurtosis is under investigation. Furthermore, the extension of the result to the MIMO case is an open problem. The characterization of the optimal transmit strategy in MIMO system under long-term power constraint is more involved since the temporal and spatial waterfilling solution cannot be easily described in closed form. Furthermore, the channel statistics act on the power allocation through the eigenvalues of the correlated Wishart matrices which have complicated expressions for the pdf [17].

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